ECE 307 – Techniques for Engineering Decisions

16. Forward Contracts

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RISK

- There are many definitions of risk; we use the conceptual definition from Webster's dictionary that ***risk is the possibility of suffering loss***
- People measure risk using a wide variety of specific metrics
- Every rational market player aims to minimize the risks faced

RISK

- **Market players use specific *financial tools* to keep their risks below a specified threshold**
- **Such actions constitute *risk management* and are carried out with financial instruments called *risk management tools***
- ***Financial derivatives* are some of the most widely-used risk management tools in financial markets**

FINANCIAL DERIVATIVES

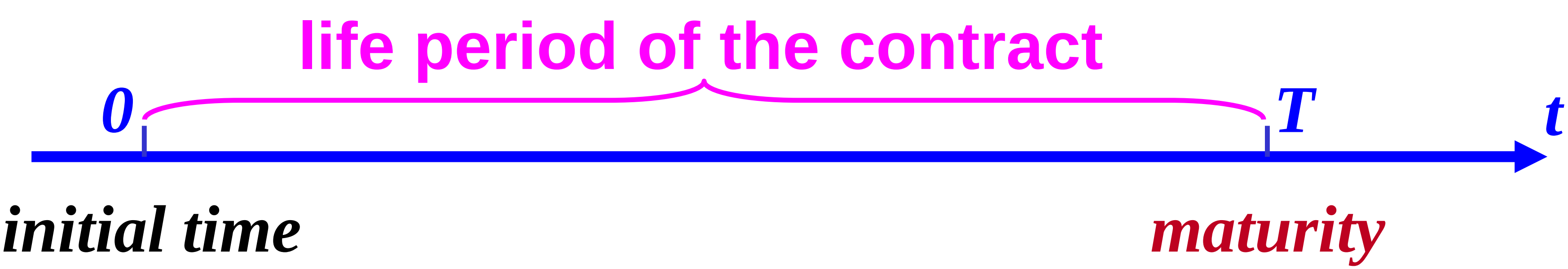
- Basic definition: a derivative is a financial tool whose value depends on the value of other, more basic underlying variables
- The basic derivatives we examine are
    - forward contracts
    - future contracts
    - options
        - puts
        - calls

EXAMPLE: A FLOUR CONTRACT

- **A farmer in Illinois and a restaurant in Wisconsin enter into a contract on January 1, 2015, under which the farmer agrees to sell 1 *ton* of flour for *$* 400 to the restaurant on September 1, 2015**
- **The contract involves two parties**
  - **the farmer is the *issuer* of the contract and holds a *short position***
  - **the restaurant is the *holder* of the contract and has a *long position***

EXAMPLE: A FLOUR CONTRACT

- **The contract is signed on January 1 for the actual sale that occurrs on September 1 and**
  - **we call January 1 the *initial time* of the contract and denote it by $t = 0$, the origin of the time line;**
  - **we call September 1 the *maturity* of the contract and denote it by $t = T$**



EXAMPLE: A FLOUR CONTRACT

- This contract is on the trading of a single specified commodity – the flour; we call the 1 *ton* of flour the *underlying asset*

- The contract provides the holder with
  - the delivery of the underlying asset at T
  - the fixed price K for the asset – the so–called *delivery price*

EXAMPLE: A FLOUR CONTRACT

- **In the absence of a forward contract, the restaurant needs to buy the flour from the spot market at an *uncertain price* to meet its needs; this price can be high so that the restaurant bears *price risks***
- **With the forward contract, the price is fixed and known and, therefore, the holder is protected from price risks**
- **This forward contract is a *physical contract* since the actual delivery of the asset is involved**

EXAMPLE: A FLOUR CONTRACT

- **We next assume the existence of a *spot market price* s_T for flour at time T so that one can buy or sell the flour at that *spot price* on the September 1 date**
- **The *flour forward contract* may be also signed as a *purely financial contract* with the flour as the underlying asset, the maturity time T of September 1, 2015, and the specification of the set of following payments:**

EXAMPLE: A FLOUR CONTRACT

- if $s_T > K$, the issuer reimburses the holder the difference $s_T - K$
- if $s_T < K$, the holder must make payment to the issuer in the amount of $K - s_T$

- These specified payments constitute the *payoff* of the financial contract
- Thus, the *net price* to the holder is the *delivery price* K and is independent of the market spot price s_T

EXAMPLE: A FLOUR CONTRACT

- Since the issuer can sell the flour in the spot market, its *net price* also equals *K*
- Therefore, this purely financial contract provides precisely the same function as the *physical contract* to both the issuer and the holder but involves no actual delivery
- Typically, many forwards are purely financial contracts that do not involve *physical deliverability*

FORWARD CONTRACTS

- **A forward contract is a binding agreement to buy or sell an asset at the designated future time at the specified price**
- **An *asset* is a general term for any good, service or commodity**
- **The buyer – holder – is said to hold a *long position* and the seller – issuer – holds a *short position***
- **The specified price is called the *delivery price***

FORWARD CONTRACTS

- **A forward contract is settled at *maturity* – the designated future time at which the purchase/ sale is consummated**
- **The *short position* delivers the asset to the *long position* in return for the cash payment of the *delivery price* times the contract amount**
- **The value of the forward contract is a function of the *market price* of the asset and its *maturity***

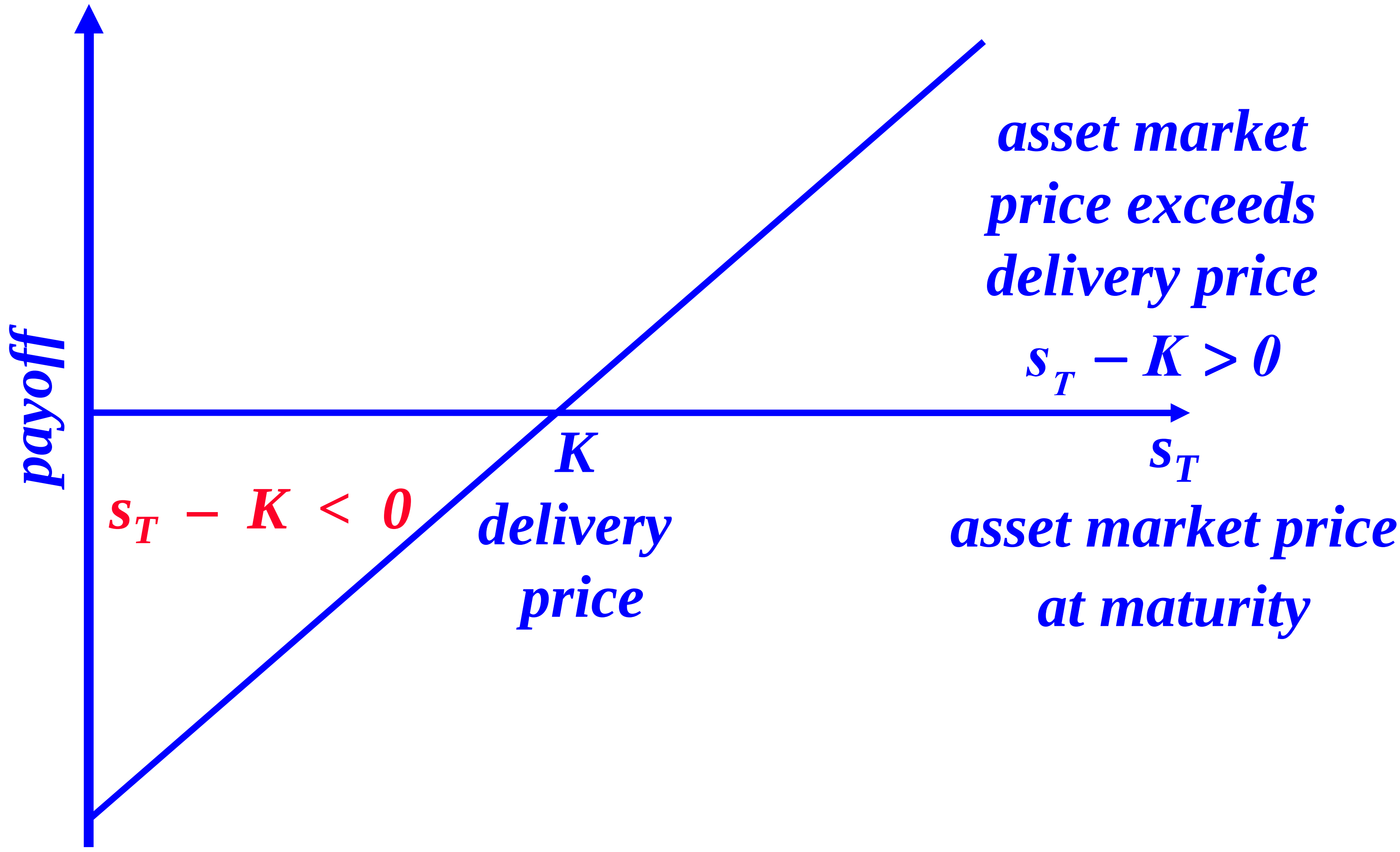
FORWARD VALUE AND PRICE

- **The value of a forward contract is *0* for both the *short* and the *long* positions at the time the contract is signed; thereafter, its value may be positive, *0* or negative**

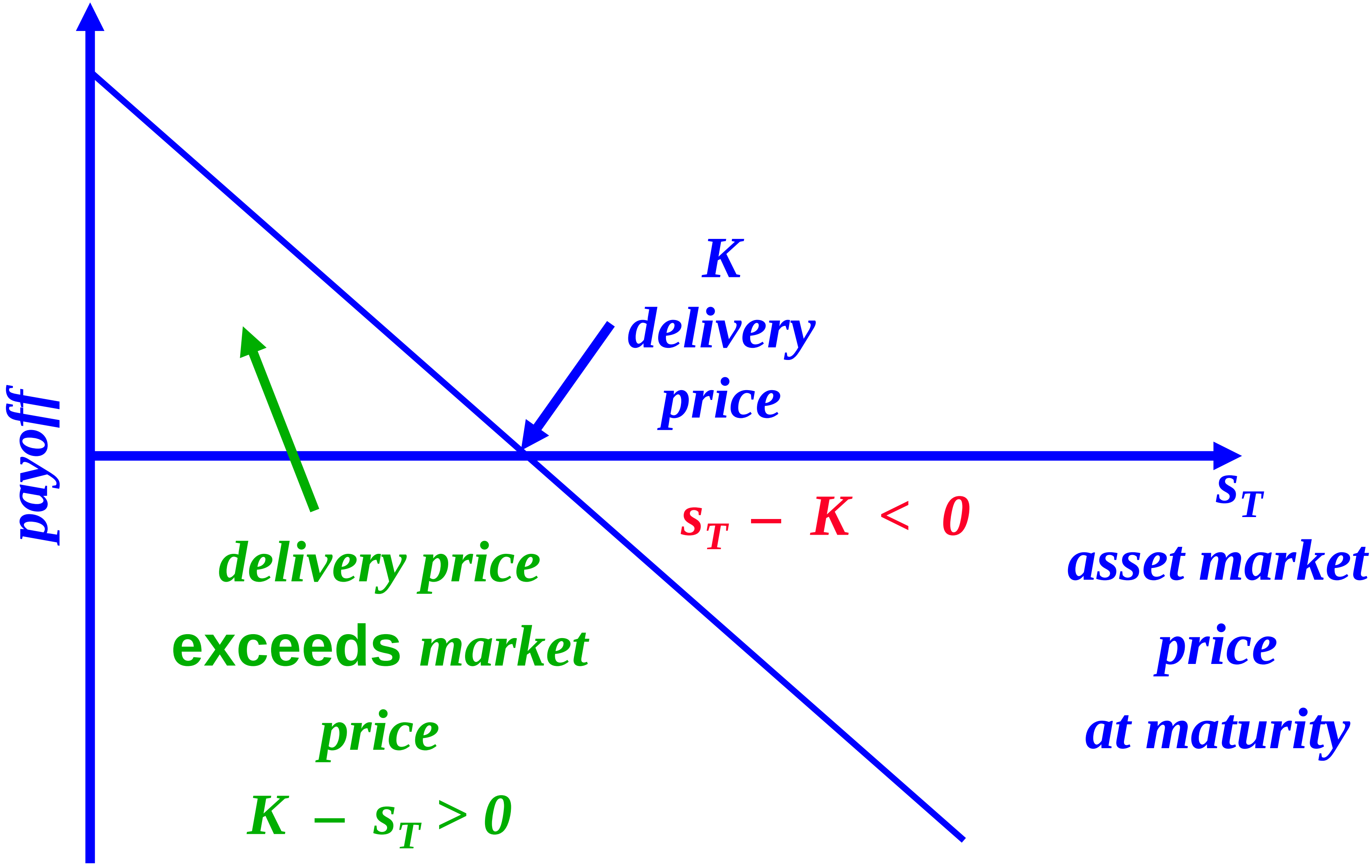
FORWARD VALUE AND PRICE

- **The *forward price* of a forward contract is the *delivery price* that makes the forward contract have *0* value at the time the contract is signed**
- **By definition, the *forward price* equals the *delivery price* at the time the contract is signed; thereafter, the *delivery price K* remains fixed but the *forward price* may change as a function of the *market price* and the *maturity* of the contract**

THE *LONG POSITION* FORWARD CONTRACT *PAYOFF*



THE *SHORT POSITION* FORWARD CONTRACT *PAYOFF*



EXAMPLE: FOREIGN EXCHANGE

May 8, 1995 spot and forward foreign exchange for *British £* and *US $*

foreign exchange	*price in US $*
spot	**1.6080**
30 – *day forward*	**1.6076**
90 – *day forward*	**1.6056**
180 – *day forward*	**1.6018**

EXAMPLE: FOREIGN EXCHANGE

- Investor signs a 90-day contract for £ 1,000,000 on May 8, 1995
- Investor pays $ 1,605,600 in 90 days and receives £ 1,000,000
- Consider two hypothetical cases:

case	s_{90}	*investor payoff* $(s_{90} - K)$ *in* $
1	1.6500	1,650,000 – 1,605,600 = 44,400
2	1.5500	1,550,000 – 1,605,600 = – 55,600

- The investor *payoff* represents the investor's total gains ($s_T - K > 0$) or total losses ($s_T - K < 0$)

FUTURES CONTRACTS

- **A *futures contract* is a *standardized forward contract* that is, typically, traded on an exchange; the exchange provides a mechanism that guarantees the contract is honored by the two parties**
- **A key aspect in which a futures contract differs from a forward contract is that an exact delivery date is not specified; typically, the futures contract specifies the delivery month**

EXAMPLE: WHEAT FUTURES CONTRACT

- **Traded on the *Chicago Board of Trade* (*CBT*)**
- **Size: 5,000 *bushels***
- **Delivery months: March, May, July, September, and December**
- **Maturity: up to 18 months in the future**
- **Quality: grades of wheat specified by *CBT***
- **Delivery locations: specified by *CBT***

FORWARD vs. FUTURES CONTRACTS

forward contract	*futures contract*
customized	*standardized*
private bilateral agreements	*publicly traded on an exchange*
the specified delivery date	*range of delivery dates*
settled at maturity (contract end)	*settled daily*
long position takes delivery; short position gets cash settlement	*typically contracts are closed out prior to maturity and do not involve delivery*

FINANCIAL DERIVATIVES : FORMAL DEFINITION

- A *financial derivative* $\mathcal{D}$ is a financial instrument that derives its values from a *related* or *underlying asset*
- Financial derivative attributes are
  - the *underlying asset* $\mathcal{S}$
  - the *maturity time* T
  - the *payoff function* $f^{\mathcal{D}}(\cdot)$

POSITIONS AND MATURITY TIME

- Two parties are involved in a financial derivative
  - the issuer: *short position*
  - the holder: *long position*
- The *maturity* is the derivative expiration time T
- The derivative may be exercised at
  - anytime $t \in \left[0, T \right]$ for *American* derivatives
  - only at $t = T$ for *European*–type derivatives
- We focus on the use of *European* derivatives: for example, in electricity T is chosen to be the time the energy is needed

THE UNDERLYING ASSETS AND ASSET MARKETS

- The derivative is written on the price movement of a traded ***underlying asset*** $\mathcal{S}$
- The ***underlying asset*** may be any good, service or variable whose value is well defined, such as a stock, a bond, a commodity, currency, or a financial contract

THE UNDERLYING ASSETS AND ASSET MARKETS

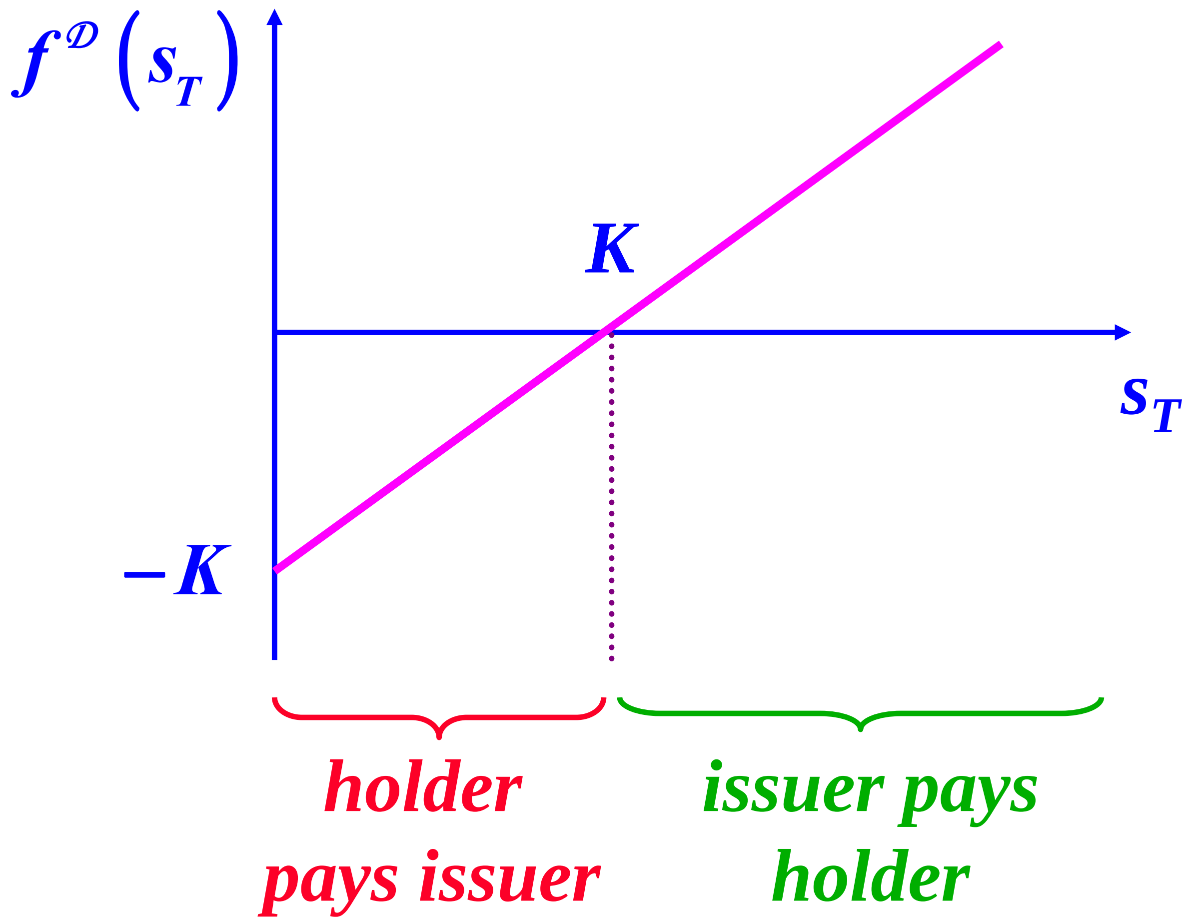
- **We assume the existence of spot markets for the underlying asset at all times during the contract life; at any time *t*, a single spot price s_t exists for the particular asset $\mathcal{S}$**
- **Short selling is allowed in the asset markets, i.e., the investor may borrow an asset from a bank and sell it, with the *obligation* to purchase the asset at a later time to return it to the bank**

PAYOFF FUNCTION OF THE DERIVATIVES

- Each derivative specifies a payment of the *payoff* from the issuer to the holder; the value of the *payoff* is expressed by the function $f^{\mathcal{D}}\left(\bullet\right)$
- The *payoff* is a function of the *underlying asset* spot price; for *European* derivatives, it is simply a function of s_T

PAYOFF EXAMPLE: THE FLOUR CONTRACT

$$f^{\mathcal{D}}\left(s_T\right) = s_T - K$$



RIGHTS AND *OBLIGATIONS*

- **In the forward flour contract example, the contract must be exercised at time T : the holder of the contract must buy the flour from the issuer who must deliver it at the time T**
- **The *payoff* of the forward is either nonnegative or negative, so that *two-sided* payments may exist**
- **Forward contracts impose *obligations* on both the issuer and the holder**

RIGHTS AND *OBLIGATIONS*

- There exist other types of derivatives, for which, the holder has the option to choose whether or not to exercise the contract
  - the holder has the *right* but not the *obligation* to exercise the contract
  - the issuer has the *obligation* to perform as the contract dictates
- Such derivatives are called *options*

OPTION CONTRACTS

- An ***option*** is a financial derivative that provides the holder the ***right*** but not the ***obligation*** to buy or sell the underlying asset at maturity at the specified strike price
- Types of options
  - ***call*** option $\mathcal{C}$: rights to buy
  - ***put*** option $\mathcal{P}$: rights to sell
  - combination of ***call*** and ***put*** options with various strike prices

EXAMPLE : ELECTRICITY *CALL*

- A generation company G issues to a broker B a *call* option $\mathcal{C}$; the option provides B the right to buy 1 MWh electricity at time T at 20 $\$/MWh$
- Attributes of the electricity *call* option
  - the *underlying asset* $\mathcal{S}$ is the 1 MWh electricity
  - the *maturity* is the time T
  - the *payoff* function is $f^{C}\left(s_{T}\right)$ with $K = 20$ $\$/MWh$

EXAMPLE : ELECTRICITY *CALL*

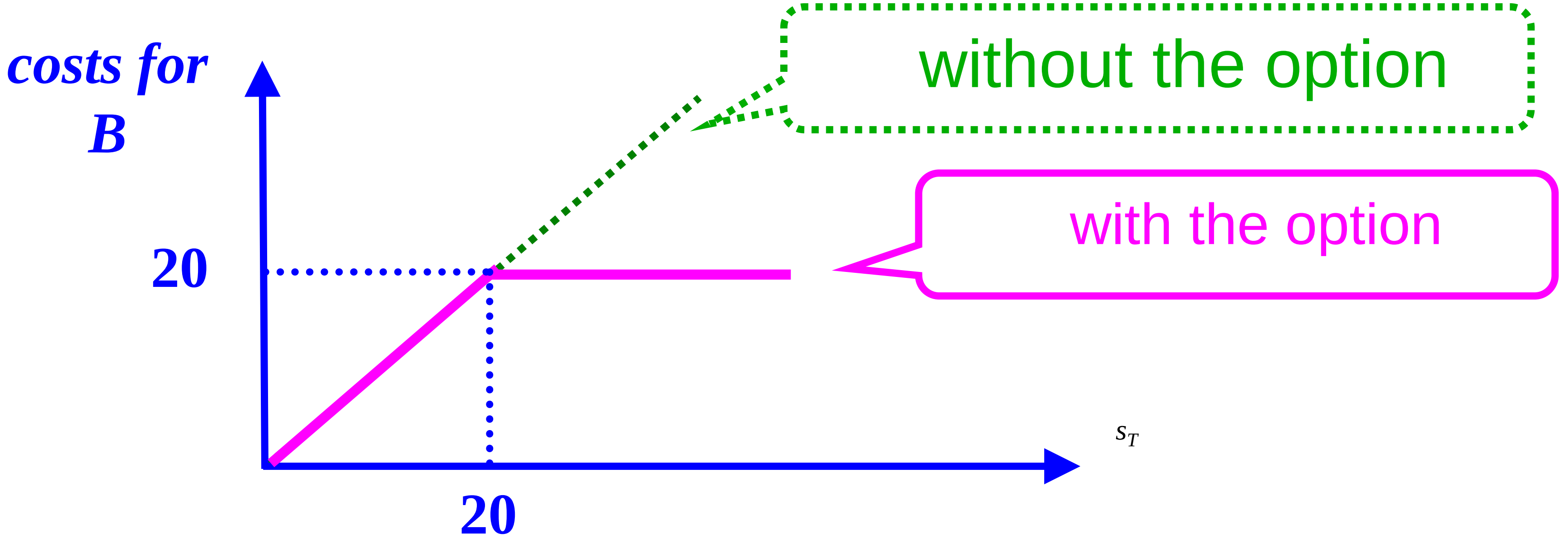
- **The contractual aspects of this option are**
  - **at the time *T*, broker *B* has the *right* but not the *obligation* to buy the 1 *MWh* from the generator *G***
  - ***G* has the *obligation* to provide the energy if requested by *B* at the *delivery price* of $K = 20$ *\$/MWh***
  - **the negotiated *strike price* 20 *\$/MWh* is totally independent of the *spot price* s_T at time *T***

EXAMPLE : ELECTRICITY *CALL*

- We assume there is 1 MWh need at time T so that B needs to buy 1 MWh to meet its demand
- We assume B is rational so that it minimizes the costs of meeting its demand
  - $s_T > 20$: B exercises the option and buys the 1 MWh from G at the fixed price $\$ 20$
  - $s_T \leq 20$: B discards the option and buys the 1 MWh from the spot market for the spot energy price s_T

EXAMPLE : ELECTRICITY *CALL*

- **The costs for *B* to purchase the 1 *MWh* is a function of s_T**



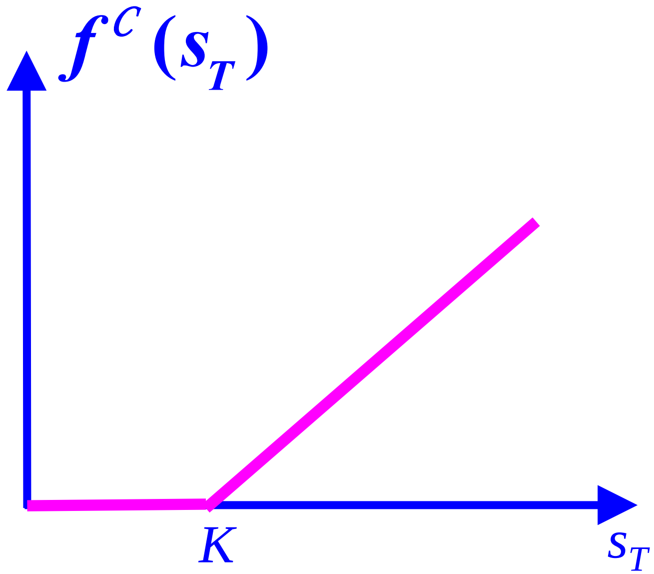
- **The *call* option protects the holder *B* from exposure to any spot price above its *strike price***

EXAMPLE : ELECTRICITY *CALL*

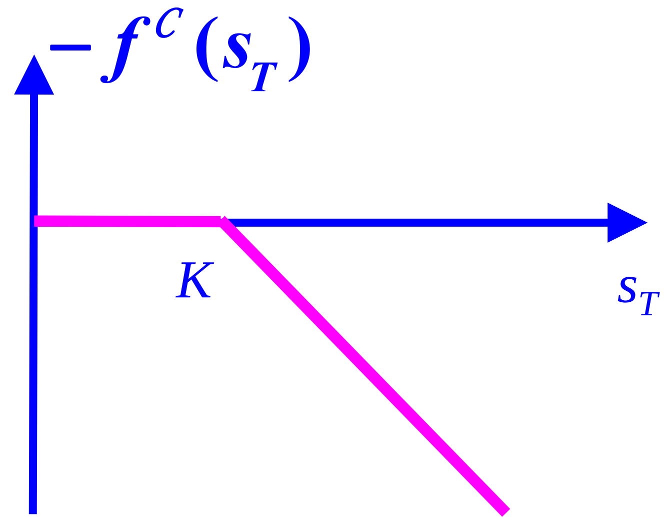
- **In financial markets, typically, the option does not involve the physical delivery of the underlying asset; instead, the following payoff is specified**
  - **if $s_T > 20$: G pays B the price difference $s_T - 20$**
  - **if $s_T \leq 20$: no payment takes place**
- **Financially, this *payoff function* provides precisely the same outcomes for G and B as if the electricity were physically delivered**

CALL OPTION *PAYOFF* DIAGRAM

$$f^{C}(s_T) = max\left\{0, s_T - K\right\}$$

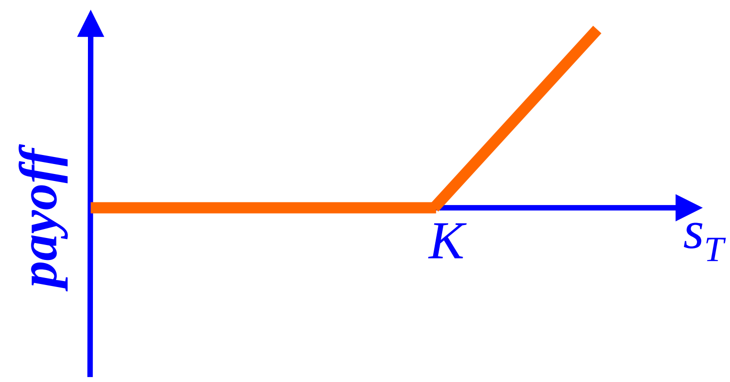
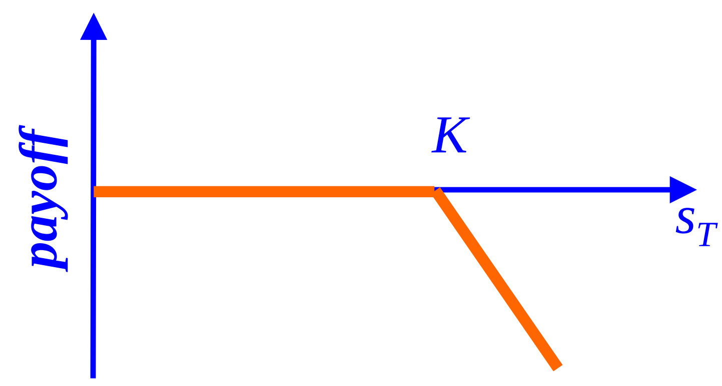


long position



short position

EUROPEAN CALL OPTION PAYOFF

position	*payoff at maturity*	
	functional form	*plot*
long	$\max\{(s_T - K), 0\}$	
short	$\min\{(K - s_T), 0\}$	

PUT OPTIONS

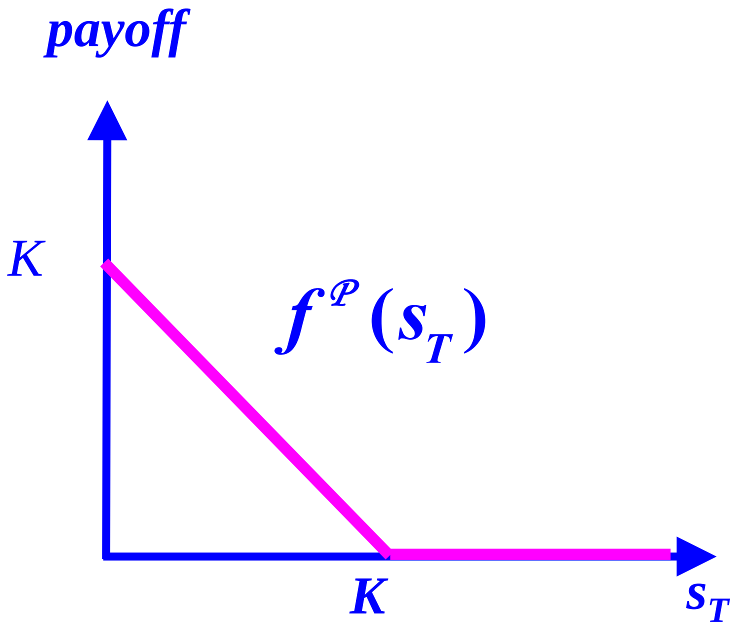
- A *put* option gives the holder the *right* to sell the underlying asset at the specified *strike price*

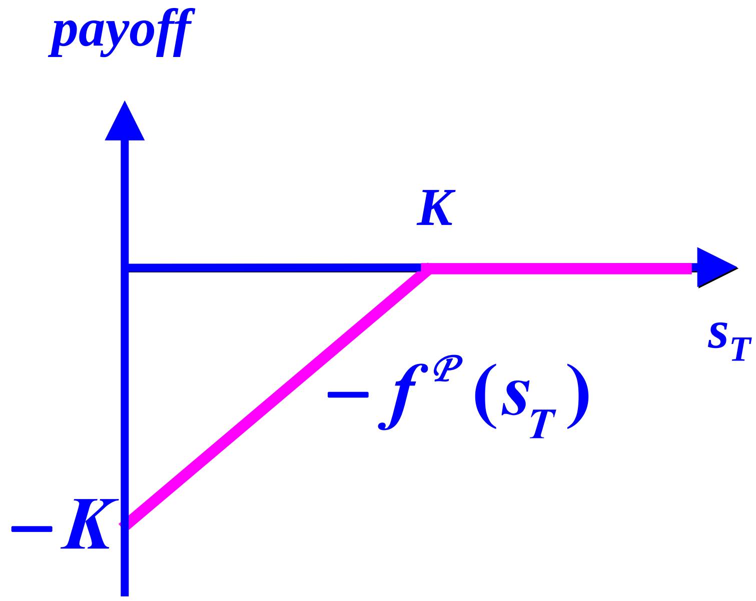
- For a *put* option $\mathcal{P}$ with the underlying asset $\mathcal{S}$, *strike price* K and *maturity* T, the *payoff* to the holder is given by

$$f^{\mathcal{P}}(s_T) = max\left\{0,\ K - s_T\right\}$$

PUT OPTION PAYOFF FUNCTION

$$f^{\mathcal{P}}(s_T) = max\left\{0, K - s_T\right\}$$

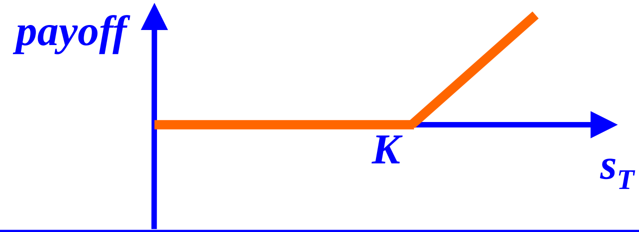
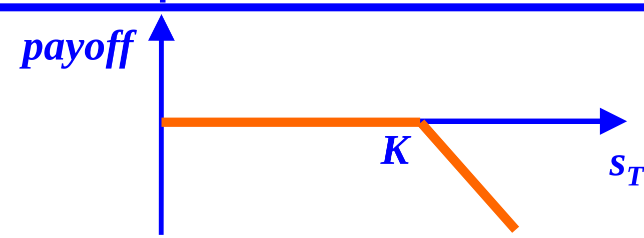
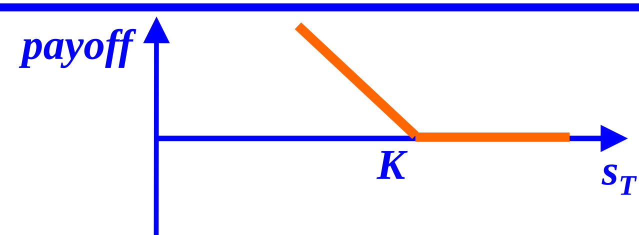
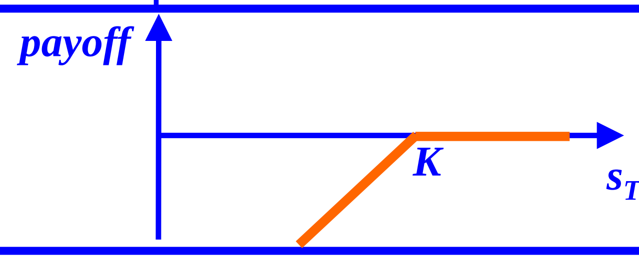




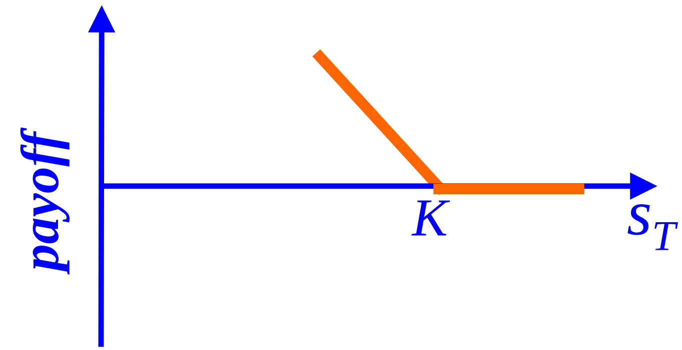
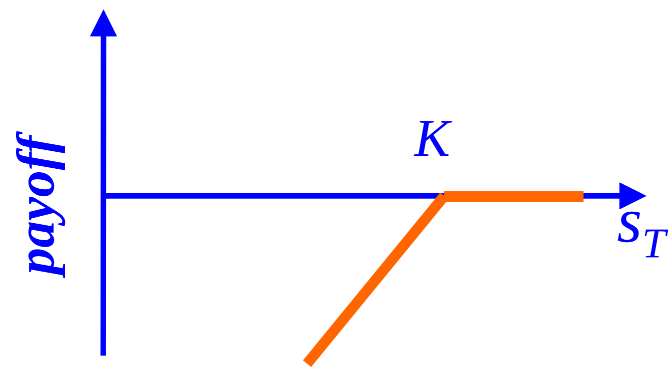
long position

short position

EUROPEAN **OPTION** *PAYOFF*

option type	position	*payoff* at maturity	
		functional form	plot
call	*long*	$max\{(s_T - K), 0\}$	
	short	$min\{(K - s_T), 0\}$	
put	*long*	$max\{(K - s_T), 0\}$	
	short	$min\{(s_T - K), 0\}$	

EUROPEAN PUT OPTION PAYOFF

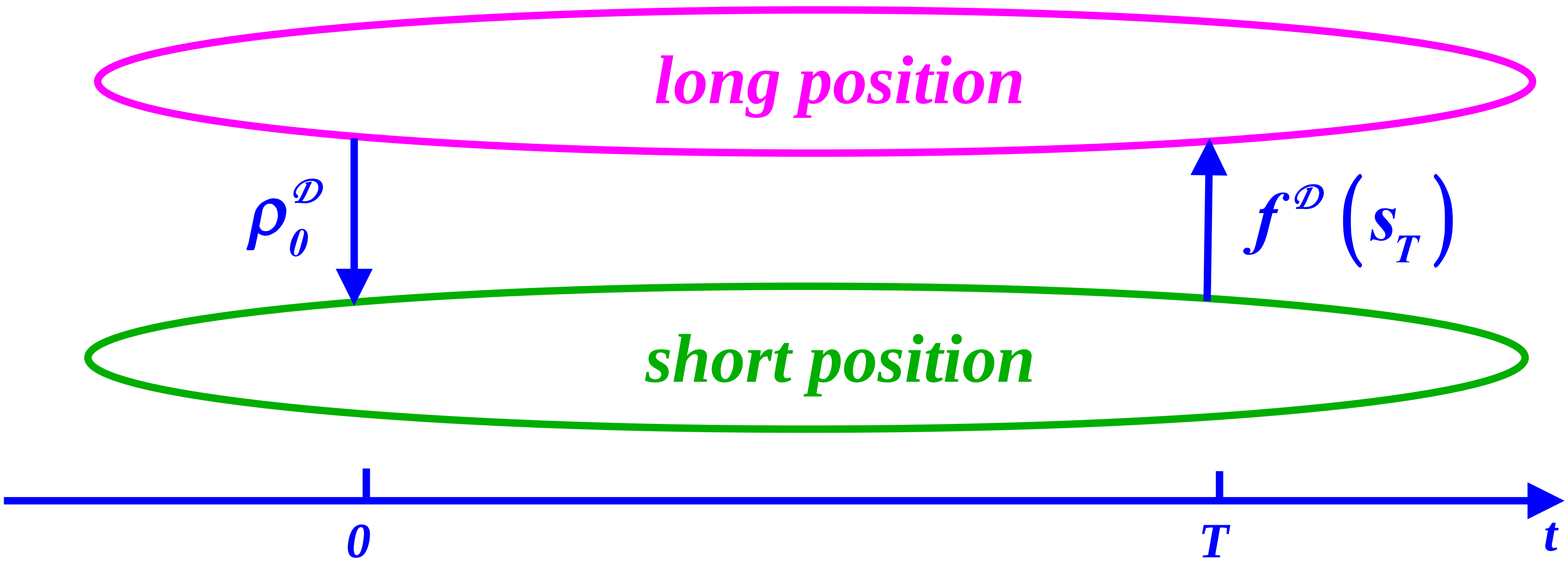
position	*payoff at maturity*	
	functional form	*plot*
long	$\max\{(K - s_T), 0\}$	
short	$\min\{(s_T - K), 0\}$	

THE OPTION *PREMIUM*

- **By definition, the *payoff* to the holder of an option contract is nonnegative, thereby providing the protection or hedge against the uncertain *spot market prices***
- **In return for such protection, the holder pays a *premium* to the issuer**
- **The *premium* is the *price* $\rho_0^{\mathcal{D}}$ of the option contract derivative $\mathcal{D}$ at $t = 0$**
- $\rho_0^{\mathcal{D}} > 0$ **when $\mathcal{D}$ is an option**

DERIVATIVE *PROFITS AND LOSSES*

- **The *profits and losses* (*P&L*) for a derivative $\mathcal{D}$, denoted by $\pi^{\mathcal{D}}$, are defined as the net cash flow into a position – *long* or *short* – during the life [*0*, *T*] of the derivative**



DERIVATIVE *PROFITS AND LOSSES*

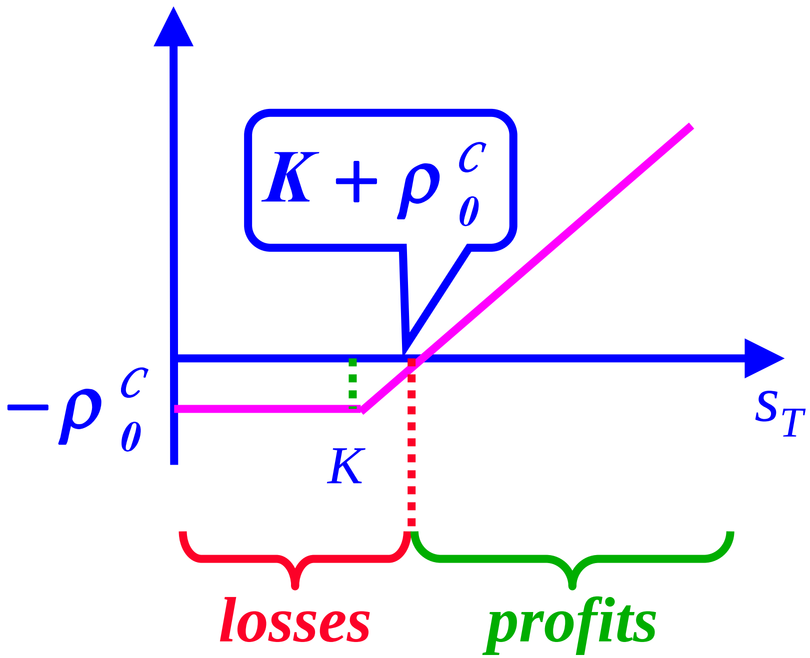
- For derivative $\mathcal{D}$ with *payoff* $f^{\mathcal{D}}\left(s_T\right)$ and *premium* $\rho_0^{\mathcal{D}}$, the *P&L* are defined as
  - *long position*: $\pi_+^{\mathcal{D}} = f^{\mathcal{D}}\left(s_T\right) - \rho_0^{\mathcal{D}}$
  - *short position*: $\pi_-^{\mathcal{D}} = - f^{\mathcal{D}}\left(s_T\right) + \rho_0^{\mathcal{D}}$
- Note that by definition

$$\pi_+^{\mathcal{D}} = - \pi_-^{\mathcal{D}} \qquad \forall \, \mathcal{D}$$

CALL OPTION *PROFITS AND LOSSES*

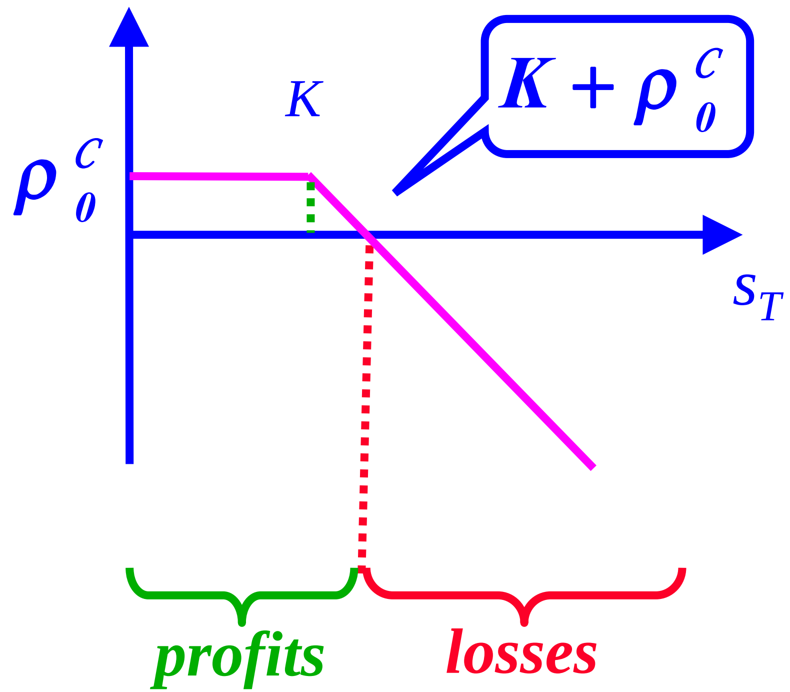
long position

$$\pi_{+}^{C} = f^{C}(s_T) - \rho_0^{C}$$



short position

$$\pi_{-}^{C} = -f^{C}(s_T) + \rho_0^{C}$$

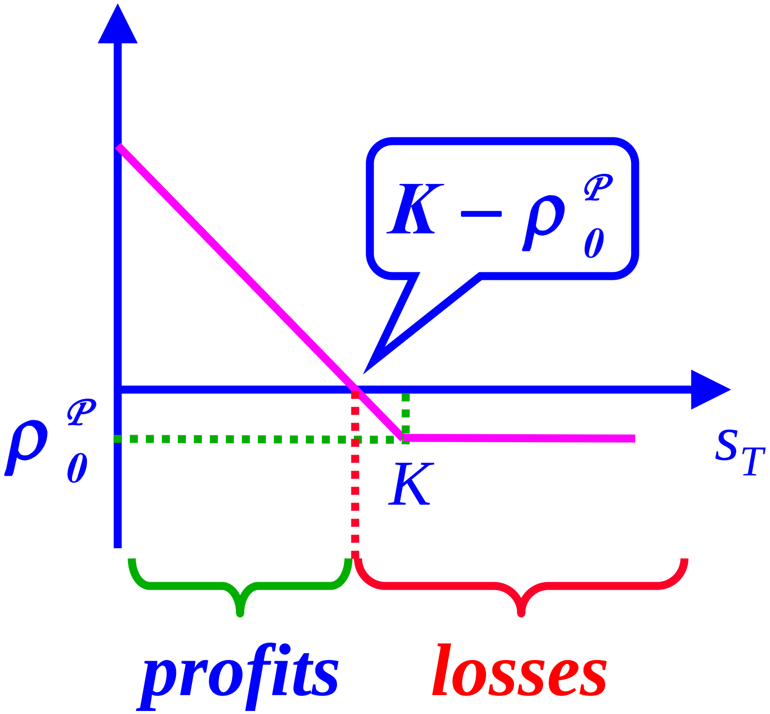


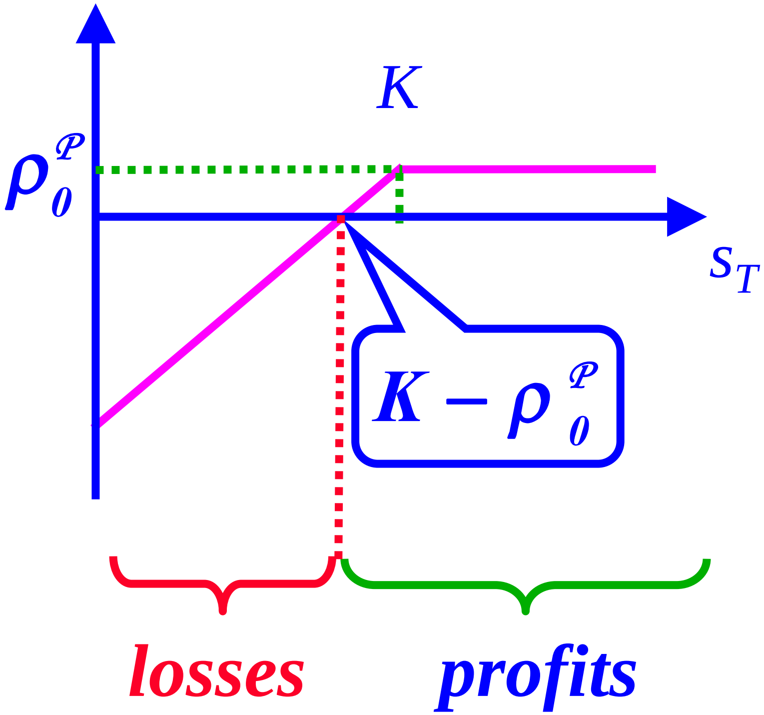
PUT OPTION *PROFITS AND LOSSES*

long position | *short position*

$$\pi_{+}^{\mathcal{P}} = f^{\mathcal{P}}(s_T) - \rho_0^{\mathcal{P}}$$

$$\pi_{-}^{\mathcal{P}} = -f^{\mathcal{P}}(s_T) + \rho_0^{\mathcal{P}}$$





HEDGING

- A *hedger* is a trader interested to reduce the risk he faces; a *hedger* uses financial derivatives to reduce exposure to movements in price
- We consider the currency exchange example: an investor needs to make £ 1,000,000 payment in 180 days and so is faced with significant foreign exchange risks in the volatile currency markets since the investor pays in *US $*

EXAMPLE : FOREIGN EXCHANGE

May 8, 1995 spot and forward foreign exchange for British *£* and *US $*

spot	**1.6080**
30 – *day forward*	**1.6076**
90 – *day forward*	**1.6018**
180 – *day forward*	**1.6056**

HEDGING STRATEGY

- Investor signs a forward contract to buy in 180 days £ 1,000,000 for *$* 1,605,600; this hedge
  - requires no initial payments
  - provides certainty for the exchange rate
  - need not ensure better outcomes

case	*rate* ($/£)	*investor's gains/losses* ($)
1	1.7000	94,400
2	1.5000	– 105,600

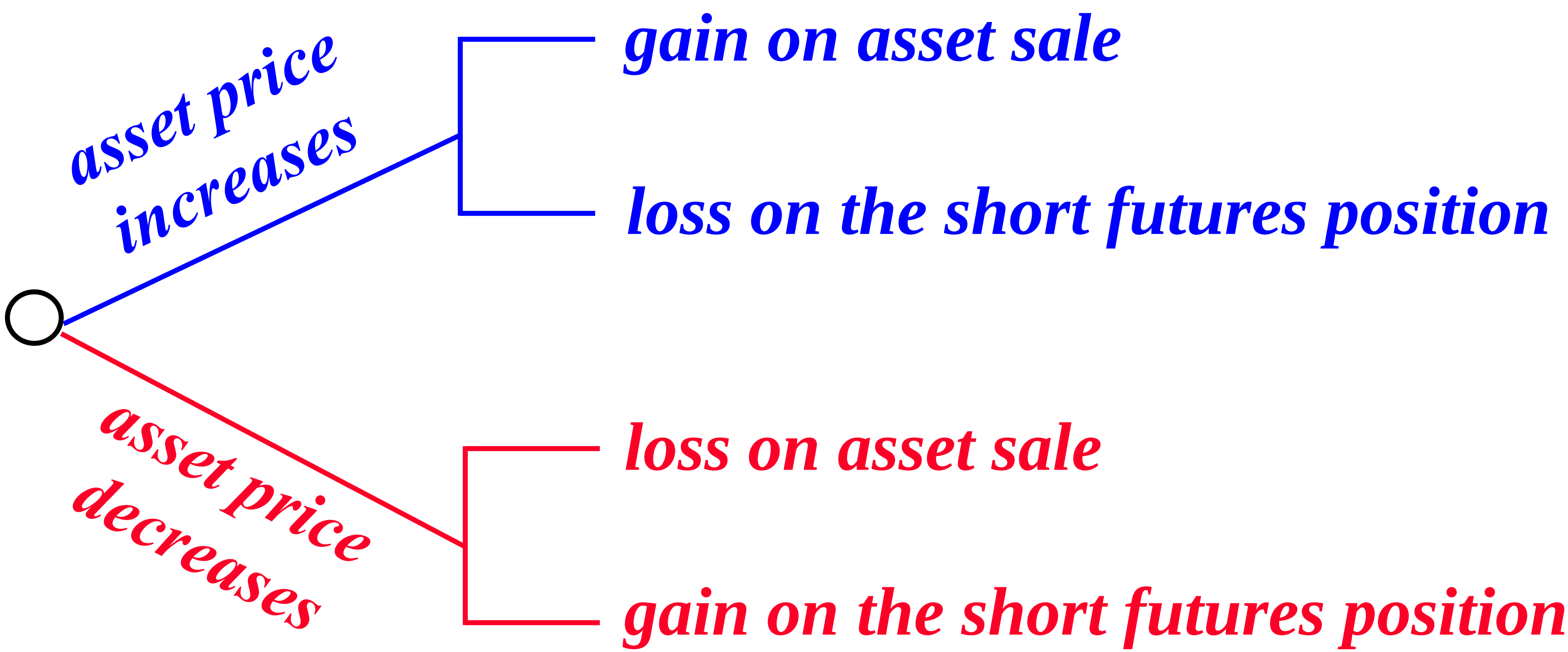
HEDGING STRATEGY

- Investor buys a *call* option to acquire £ 1,000,000 at the exchange rate of 1.6000; this hedge
  - requires an initial outlay for the *call* option premiums
  - provides insurance to the investor against adverse exchange rate movements and benefits from favorable movements

case	*exchange rate*	*investor's action*
1	> 1.6	exercise option
2	< 1.6	buy £ in market

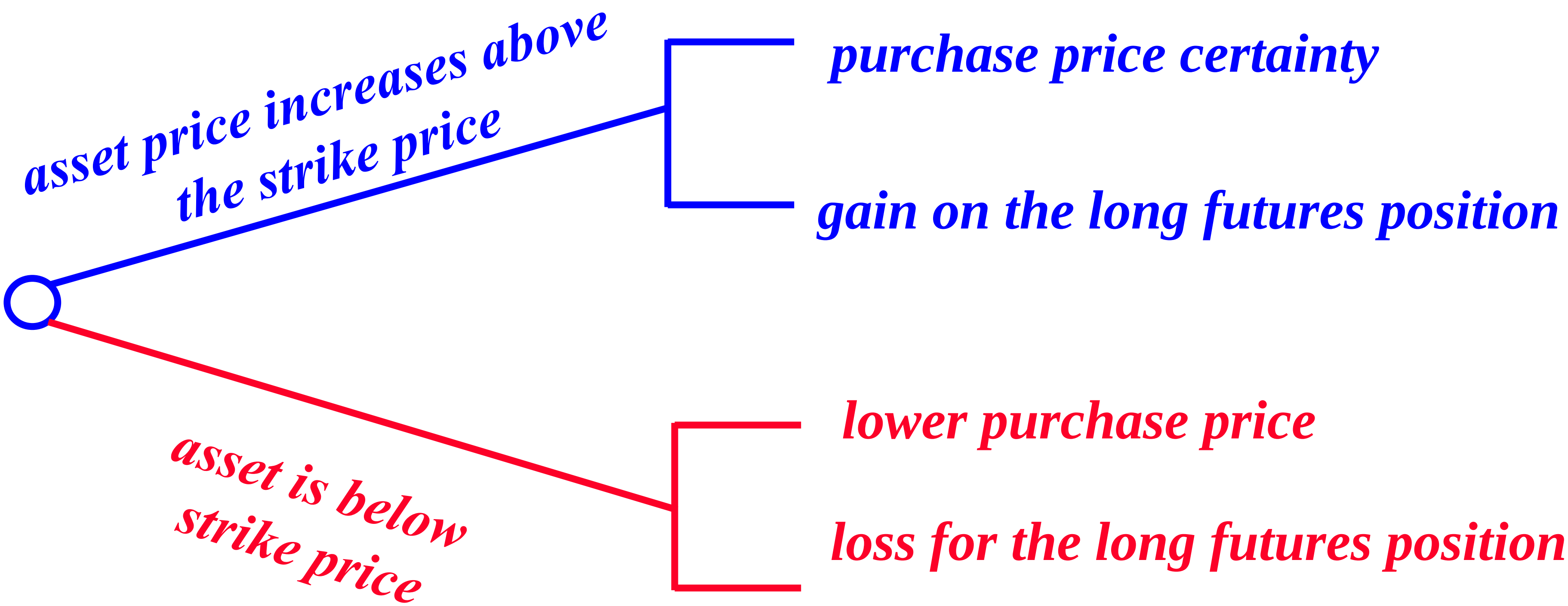
USING *FUTURES* FOR *HEDGING*

- **An entity that sells an asset at some given future time can hedge by taking a *short futures position*; this is a *short hedge***



USING *FUTURES* FOR *HEDGING*

- **An entity that wishes to buy an asset at some future time can hedge by taking a *long futures position*; this is a *long hedge***



USING *FUTURES* FOR *HEDGING*

- Futures hedging does not necessarily improve the overall financial outcome; in fact, on the average the outcome is worse 50% of the time
- Futures hedging, however, reduces risk since it provides price certainty

EXAMPLE : *SHORT HEDGE*

- We consider a generator who is planning the sale of its 1 *MWh* energy production at an hour *T* at a future time
- We assume the production costs for the 1 *MWh* energy are at 20 *$ / MWh*
- We assume the energy spot price s_T at hour *T* has the following discrete distribution

$$s_T = \begin{cases} 18 \;\; \$ / MWh & \text{with probability } 0.5 \\ 26 \;\; \$ / MWh & \text{with probability } 0.5 \end{cases}$$

EXAMPLE : *SHORT HEDGE*

- If the generator sells its energy directly in the spot market, its profits are

$$\pi_T = \begin{cases} -2\ \$ & \textit{with probability } 0.5 \\ 6\ \$ & \textit{with probability } 0.5 \end{cases}$$

- The generator suffers a loss in the case the spot price is $18\ \$/MWh$ since his marginal costs are $20\ \$/MWh$
- The generator may protect himself from such a loss by taking a *short futures position*

EXAMPLE : *SHORT HEDGE*

- If the generator sells a *futures* contract $\mathcal{A}$ on 1 *MWh* electricity with the *maturity T* and the *delivery price* $K = 22\ \$ / MWh$, the generator receives the net *payoff* at time *T* of

$$f^{\mathcal{A}}(s_T) = -(s_T - K) = \begin{cases} 4\ \$ & \text{if} \quad s_T = 18\ \$ / MWh \\ -4\ \$ & \text{if} \quad s_T = 26\ \$ / MWh \end{cases}$$

- The generator's net profits then become

$$\pi_T^{net} = \pi_T + f^{\mathcal{A}}(s_T) = \begin{cases} 2\ \$ & \textit{with probability } 0.5 \\ 2\ \$ & \textit{with probability } 0.5 \end{cases}$$

EXAMPLE : *SHORT HEDGE*

- **So, as the holder of the *short futures position*, the generator's *net profits* become independent of the *spot price*; the generator gains in either case**
- **In this way, we say the generator's position is *fully hedged***

EXAMPLE : *LONG HEDGE*

- We consider a 1 MW load planning its energy purchases for the hour T at a future time
- We assume the energy spot price s_T at hour T has the following distribution

$$s_T = \begin{cases} 18 \quad \$ / MWh & \quad \textit{with probability } 0.5 \\ \\ 26 \quad \$ / MWh & \quad \textit{with probability } 0.5 \end{cases}$$

EXAMPLE : *LONG HEDGE*

- **If the load purchases its energy directly in the spot market, its costs are**

$$c_T = \begin{cases} 18\ \$ & \textit{with probability } 0.5 \\ \\ 26\ \$ & \textit{with probability } 0.5 \end{cases}$$

- **The load faces uncertainty in the supply costs**
- **The load may get price certainty by taking a *long futures position***

EXAMPLE : *LONG HEDGE*

- If the load holds a *long futures position* $\mathcal{A}$ on 1 *MWh* energy with the *maturity* T and the *delivery price* $K = 22\ \$ / MWh$ at T, the load receives the net *payoff*

$$f^{\mathcal{A}}(s_T) = s_T - K = \begin{cases} -4\ \$ & \text{if} \quad s_T = 18\ \$ / MWh \\ 4\ \$ & \text{if} \quad s_T = 26\ \$ / MWh \end{cases}$$

- The load's net costs then become

$$c_T^{net} = c_T - f^{\mathcal{A}}(s_T) = \begin{cases} 22\ \$ & \textit{with probability } 0.5 \\ 22\ \$ & \textit{with probability } 0.5 \end{cases}$$

EXAMPLE : *LONG HEDGE*

- **In other words, by holding the *long futures position*, the load's net costs are independent of the spot prices; the load gets the price certainty**

- **In this way, we say the load's position is *fully hedged***

SPECULATION

- **A speculator takes a position in a market by betting that either that a price increases or a price decreases**
- **A speculator may purchase the asset on the spot market and rely on**
  - **future spot markets**
  - **use *forward contracts* with a higher level of leverage**
  - **use *options* for additional leverage**

SPECULATION

- **In the *sterling exchange* example, the investor can speculate and take a long position in a 180-day forward contract on sterling: suppose the speculator buys a 180-day forward contract at conversion rate of 1.6056 and the exchange rate rises to 1.7000, then he makes a profit of *$* 94,440**

EXAMPLE : SPECULATION USING OPTIONS

- **A stock price is *$* 43 and an investor is betting that the price will rise and buys *call* options with a *strike price* of *$* 48 at *$* 1 per option**
- **If the price fails to go above *$* 48 during the life of the option, the speculator loses *$* 1 per option**
- **If the price rises to *$* 55, the speculator realizes net *profits* of *$* 6 per option representing a 600% gain on the original investment of the premium**

ARBITRAGE

- Arbitrageurs lock in *riskless profits* by undertaking transactions in two or more markets
- As an example we consider a situation where a stock is traded on stock exchanges both in NY and London: suppose stock price is *$* 172 in NY and £ 100 in London when the exchange rate is *$* 1.750 per £:
  - arbitrageur buys 1000 shares in NY and sells them in London to obtain a *risk free profit* of *$* 3,000

ARBITRAGE

- such arbitrage opportunities do not last very long because as arbitrageurs buy more stocks in NY, the law of supply and demand causes the price to rise and, similarly, as they sell more stock in London, the £ price will decrease so that in a short time the prices become the same at the current exchange rate